SE312

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S.E. Mechanical (Sem-III) (Revised Course 2016-2017)
EXAMINATION Nov/Dec 2019
Engineering Mathematics and Numerical Techniques

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:

1) Answer five questions. Any two from part-A, any two from part-B and any one from part-C.
2) Assume suitable data, if necessary.
3) Figures to right indicate full marks.

Part A

Q.1

a) Prove the following **(6 Marks)**

i) If A is a square matrix then $A + A^T$ is symmetric and $A - A^T$ is skew-symmetric matrix.

ii) If A is square matrix then $adjA^T = (adjA)^T$

b) Find the rank of the following by reducing it into its normal form. **(8 Marks)**

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

c) Find the Laplace transform of **(6 Marks)**

i) $t\, e^{2t} sin5t$

ii) $\frac{e^{-t} - Cost}{t}$

Q.2

a) Find the values of α for which the system has a unique solution. Solve in each case. **(7 Marks)**

$x + y + z = 1$

$x + 2y + 4z = \alpha$

$x + 4y + 10z = \alpha^2$

b) Prove that **(6 Marks)**

i) $If\ L\{f(t)\} = F(s)$ then $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$

ii) $If\ L\{f(t)\} = F(s)$, then $L\{f'(t)\} = sL\{f(t)\} - f(0)$

c) Find $L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$ by using Convolution theorem **(7 Marks)**

Q.3

a) Examine the following system of vectors for Linear Dependence. **(6 Marks)**

$X_1=(1, 1, 1, 3)$; $X_2=(1, 2, 3, 4)$; $X_3=(2, 3, 4, 7)$

b) Diagonalize the following matrix **(8 Marks)**

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$$A = \begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}$$

c) Find the solution of the differential equation using Laplace Transforms: **(6 Marks)**
$(D^2 + 5D + 5)y = e^{-t}; y(0) = 0\,; y'(0) = 1$

Part B

Q.4 a) Find Fourier series of $f(x) = x + x^2$ for $-\pi < x < \pi$. **(8 Marks)**
Hence deduce that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

b) Find the real root of the equation **(6 Marks)**
$x \sin x + \cos x = 0$ using Bisection method up to the sixth approximation.

c) **Calculate $f(410)$ from the following table.** **(6 Marks)**

x	100	150	200	250	300	350	400
y	10.6	13.0	15.0	16.8	18.4	19.9	21.3

Q.5 a) Find Fourier Transform of the following function of **(6 Marks)**

$$f(x) = 1, |x| < a$$
$$= 0, |x| > a$$

b) Using Lagrange's interpolation formula find f (9) from **(6 Marks)**

x	5	7	11	13	17
y	150	392	1452	2366	5202

c) State and prove Newton's forward interpolation formula. **(8 Marks)**

Q.6 a) Solve the integral equation. **(7 Marks)**

$$\int_0^\infty f(x)\cos(sx)dx = \begin{cases} 1 - s, & 0 \le s \le 1 \\ 0, & s > 1 \end{cases}$$

b) Evaluate $\sqrt[4]{32}$ using Newton Raphson method correct to 3 places of decimals. **(6 Marks)**

c) Obtain Fourier Sine series for x^2 in the interval $0 < x < 2$ **(7 Marks)**

Part C

Q.7 a) Verify Cayley- Hamilton Theorem and hence obtain the inverse of the matrix **(6 Marks)**

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

b) Evaluate: $\int_0^\infty t\, e^{-2t} \sin t \, dt$ **(6 Marks)**

c) Use sterling's interpolation formula to compute y at x = 3.8 form the given table. **(8 Marks)**

x	1		2	3	4	5	6	7
y	8.06		11.06	14.42	18.32	22.95	28.28	34.46

Q.8 a) If $F(s)$ is Fourier Transform of $f(x)$, then prove that **(8 Marks)**

i) $F\{f(x)\cos ax\} = \frac{1}{2}[F(s+a) + F(s-a)]$

ii) $F\{e^{iax} f(x)\} = F(s+a)$

b) Obtain the inverse Laplace transform of: $\frac{1}{2} log\left(\frac{s^2+b^2}{s^2+a^2}\right)$ **(6 Marks)**

c) State and prove Newton's divided difference formula. **(6 Marks)**

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