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S.E. (Information Technology) (Sem-III) (Revised Course 2016-2017)  
 EXAMINATION Nov/Dec 2019  
 Applied Mathematics -III

[Duration : Three Hours]

[Total Marks : 100]

Instructions:-

- 1) Attempt five questions, any two questions each from Part-A and Part-B and one from Part-C.
- 2) Assume suitable data, if necessary.
- 3) Figures to the right indicate full marks.

PART- A

(Answer any two questions)

Q.No.1 a) Find the rank of the following matrix (7)

$$A = \begin{pmatrix} 1 & 3 & 3 & -1 \\ 2 & -2 & 3 & 1 \\ 3 & 1 & 6 & 0 \\ 1 & 3 & 3 & -1 \end{pmatrix} \text{ and hence reduce it into its Normal form}$$

b) Test the consistency of the system of equations (6)

$$\begin{aligned} x - 3y + 2z &= 1 \\ 2x + y - 2z &= 2 \\ 2x + 4y - 6z &= -2 \end{aligned}$$

Find the solution if it is consistent

c) The waiting time (in minutes) is between two successive speeders spotted by a radar is continuous random variable with probability distribution function. (7)

$$f(x) = \begin{cases} 0, & x < 0 \\ 8e^{-8x}, & x > 0 \end{cases} \text{ Find the Probability of waiting less than 12 minutes between successive speeders.}$$

Q.No.2 a) Prove that Eigen values of an orthogonal matrix are either 1 or -1 (5)

b) Find the minimal polynomial of the matrix. (8)

$$A = \begin{bmatrix} 4 & 2 & 3 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$$

c) The probability density function of a continuous random variable is (7)

$$f(x) = \begin{cases} Kx(1-x)e^x & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases} \text{ find the value of K and hence find the mean and variance of the distribution.}$$

Q.No.3

- a) Verify Cayley Hamilton theorem for  $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ 1 & -1 & 3 \end{bmatrix}$  (6)
- b) The mean height of 500 male students in a certain college is 151 cm and the standard deviation is 15 cm. Assuming the height are normally distributed, find how many students have heights between 120 cm and 155 cm. (7)
- c) Find the Cumulative distribution function of the following function (7)
- $$f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$
- and hence find
- $P(0.5 < x < 1)$

## PART- B

(Answer any two questions)

Q.No.4

- a) Find the Laplace Transform of (6)
- i)  $t^2 \cos^2 t$  ii)  $t \sin(2t) \cos 3t$
- b) If  $L[f(t)] = F(s)$  then show that (6)
- i)  $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(u)du$  ii)  $L[f'(t)] = sF(s) - f(0)$
- c) Find the Fourier Cosine and Sine Transform of  $x^{n-1}$ . (8)

Q.No.5

- a) Use Laplace Transform to solve  $y'' - 3y' - 4y = 2e^{-t}$ ,  $y(0) = 0 = y'(0)$  (8)
- b) Find the inverse Laplace Transform of (6)
- i)  $\tan^{-1}(s+1)$  ii)  $\frac{s}{(s^2+2^2)^2}$
- c) Solve for  $f(x)$ , given that  $\int_0^\infty f(x) \sin x dx = e^{-2s}$ ,  $s > 0$  (6)

Q.No.6

- a) Find the Z-transform of the following (3+4)
- i)  $4^n \cdot \cos\left(\frac{n\pi}{2}\right)$  ii)  $\frac{2}{(n+1)(n-1)}$
- b) Using Convolution Theorem, find the inverse Laplace transform of  $\frac{2s}{(s-1)(s^2+1)}$  (6)
- c) Find the inverse Z-Transform of  $\frac{2z^2-z}{(z-1)(z^2+4)}$  (7)

## PART-C

(Answer any one questions)

Q.No.7

- a) Find the Eigen values and Eigen vectors of (8)

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

- b) Using Laplace transforms method, solve the integral equation (7)

$$y(t) = e^{3t} + 4 \int_0^t y(u) du$$

- c) Define Uniform distribution. Find the mean and variance of Uniform distribution. (5)

Q.No.8

- a) If
- $F[f(x)] = F(s)$
- is Fourier transform of
- $f(x)$
- , show that (6)

$$i) F[e^{iax} f(x)] = F(s + a) \quad ii) F[f'(x)] = -i s F(s) \text{ if } f(x) \rightarrow 0 \text{ as } x \rightarrow \infty$$

- b) Solve the difference equation
- $y_{n+2} + 3y_{n+1} + 2y_n = 4^n$
- using Z-Transform method, given
- $y(0) = 0, y_1(0) = 0$
- (8)

- c) Determine the values of
- $\lambda$
- and
- $\mu$
- for which the following system of equations (6)

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + \lambda z = \mu$$

Has

- i) No solution
- ii) Unique solution
- iii) Infinite number of solutions.