

Total No. of Printed Pages:2

S.E.(Electronics & TC) / Electronics & Comm Engg Semester-IV (Revised Course 2007-08)  
 EXAMINATION MAY/JUNE 2019  
 Applied Mathematics - IV

[Duration : Three Hours]

[Total Marks : 100]

- Instructions :**
- 1) Attempt any five questions with at least one from each module.
  - 2) Assume suitable data if required.

**MODULE - I**

- Q.1 (a) Reduce the differential equation  $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + (x^2 - n^2 + 1)y = 0$  to Bessel's differential equation. 07
- (b) Prove that  $\int_0^1 x J_n^2(\alpha x) dx = \frac{1}{2} J_{n+1}^2(\alpha)$  07
- (c) Prove that  $J_0^2(x) + 2(J_1^2(x) + J_2^2(x) + J_3^2(x) + \dots) = 1$  06
- Q.2 (a) State and Prove the generating function of  $J_n(x)$  07
- (b) Show that  $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin\theta) d\theta$  07
- (c) Expand  $f(x) = 1 - x^2, 0 \leq x \leq 1$  in a Fourier Bessel's series in terms of Bessel's function of order 2 06

**MODULE - II**

- Q.3 (a) State and Prove Rodrigues formula. 07
- (b) Show that  $4x^3 - 2x^2 - 3x + 8 = \frac{8}{5}P_3(x) - \frac{4}{3}P_2(x) - \frac{3}{5}P_1(x) + \frac{22}{3}$  05
- (c) Prove the following (i)  $P_{n+1}(x) - P_{n-1}(x) = n[P_{n-1}(x) - xP_n(x)]$  08  
 (ii)  $(1 - x^2)P_n'(x) = n[P_n(x) - xP_n'(x)]$
- Q.4 (a) Prove the following 06  
 (i)  $P_n(1) = 1$   
 (ii)  $P_n(-1) = (-1)^n$
- (b) Prove that  $\int_{-1}^1 (1 - x^2)(P_n(x))^2 dx = \frac{2n(n+1)}{(2n+1)}$  08
- (c) Express the polynomial  $f(x) = x^3 + 2x^2 - x - 3$  in terms of the Legendre's polynomial. 06

MODULE-III

Q.5 (a) Evaluate:  $\int_C \frac{7z-1}{z^2-3z-4} dz$  where C is the ellipse  $x^2 + 4y^2 = 4$

07

(b) Expand  $f(z) = \cos z$  as a Taylor's series about the point

06

(i).  $z = \frac{\pi}{3}$  (ii).  $z = \frac{\pi}{4}$

(c) Find the Taylor's series and Laurent's series which represents the function

07

$\frac{z}{(z+1)(z+2)}$  in

(i)  $1 < |z| < 2$

(ii)  $|z+1| < 1$

Q.6 (a) State and prove Cauchy integral theorem

08

(b) Find the residues at  $z = 0$  of the functions

06

(i).  $f(z) = e^{1/z}$

(ii)  $f(z) = z \cos\left(\frac{1}{z}\right)$

(c) Evaluate.  $\int_C \frac{2z-3}{z^3-3z^2+4} dxz$ . Where C is the circle  $|z| = \frac{3}{2}$

06

MODULE-IV

Q.7 (a) Evaluate  $\int_0^\infty \frac{\cos ax}{x^2+1} dx$ ,  $a > 0$

06

(b) Using Cauchy's residue theorem, evaluate  $\int_0^{2\pi} \frac{\cos^2 3\theta}{5-4\cos 2\theta} d\theta$

07

(c) Evaluate:  $\int_{-\infty}^{+\infty} \frac{x^2}{(x^2+4)(x^2+9)} dx$ , using Contour integration.

07

Q.8 (a) A tightly stretched string of length  $l$  has its end fastened at  $x = 0$  and  $x = l$ . At time  $t = 0$  the string was released from rest with the displacement  $f(x) = k(x)(l-x)$ . Find the displacement at a distance  $x$  from one end at any time  $t$ .

10

(b) Derive the On-dimensional heat equation.

10