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S.E. (Electronics & TC/Electronics & Comm Engg) Semester- IV (Revised Course 2007-08)
EXAMINATION Aug/Sept 2019
Applied Mathematics-IV

[Duration : Three Hours]

[Max. Marks : 100]

Instructions:

- 1) Attempt **any five** questions with at least one from each module.
- 2) Assume suitable data if required.

MODULE -I

1 a) Find the general solution of the differential equation: 07
 $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0$

b) Prove the following

$$1) \frac{d}{dx} (x J_n(x) J_{n+1}(x)) = x (J_n^2(x) - J_{n+1}^2(x))$$

$$2) \int J_3(x) dx + J_2(x) + \frac{2}{x} J_1(x) = 0$$

$$3) 4J_0'''(x) + 3J_0'(x) + J_3(x) = 0$$

13

2 a) State and prove the orthogonality of Bessel's function 08

b) Show that $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin \theta) d\theta$ 07

c) Expand $f(x) = 1 - x^2, 0 \leq x \leq 1$ in a Fourier Bessel's series in terms of Bessel's function of order 2. 05

MODULE -II

3 a) State and prove Rodrigues formula 07

b) Prove that $p'_{n+1}(x) + p'_n(x) = p_0(x) + 3p_1(x) + 5p_2(x) + \dots + (2n+1)p_n(x)$ 07

c) Prove the following

$$i) P_{2n}(x) = (-)^n \frac{(2n)!}{2^n (n!)^2}$$

$$ii) (1-x^2) P'_n(x) = n [P_{n-1}(x) - x P_n(x)]$$

06

- 4 a) Show that the Legendre's Polynomial $\{P_n(x)\}$ form an orthogonal set over $[-1, 1]$ 07
- b) Prove that $\int_{-1}^1 x^2 \{2P_{n+1}(x)P_{n-1}(x) - P_n^2(x)\} dx = \frac{2}{(2n-1)(2n+1)(2n+3)}$ 07
- c) If $f(x) = x, x \in [-1, 1]$, show that $x = \frac{1}{2}P_0(x) + \frac{5}{8}P_2(x) - \frac{3}{16}P_4(x) + \dots$ 06

MODULE -III

- 5 a) State and prove Cauchy integral formula. 08
- b) Evaluate. $\int_C \frac{2z-3}{z^3-3z^2+4} dxz$. where C is the circle $|z| = \frac{3}{2}$ 06
- c) Test for singularity of $f(z) = \frac{1}{z^2+1}$ and hence find the corresponding residues 06
- 6 a) Determine the singularities of the function $f(z) = \frac{(z^2-2z)}{(z+1)(z^2+4)}$ and compute the residues at the singularities. 07
- b) Find the Laurent series expansion of $f(z) = \frac{7z-2}{z(z^2-z-2)}$ valid in $1 < |z+1| < 3$ 06
- c) Using Cauchy's integral theorem , evaluate 07

$$\int_C \frac{z}{(z-1)(z-2)^2} dz \text{ where } c: |z-2| = \frac{1}{2}$$

MODULE- IV

- 7 a) State and prove Lowville's theorem 06
- b) Using contour integration , evaluate $\int_0^{2\pi} \frac{d\theta}{13+5 \sin \theta}$ 07
- c) Using Contour integration evaluate $\int_{-\infty}^{+\infty} \frac{x}{(x+1)(x^2+1)} dx$ 07
- 8 a) Derive the solution of one – dimensional heat equation. 10
- b) Derive the one – dimensional wave equation. 10