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S.E.(Computer) (Semester- III) (Revised Course 2007-08) EXAMINATION MAY/JUNE 2019
Applied Mathematics - III

[Duration : Three Hours] **[Max Marks : 100]**

Instruction:-
Please check whether you have got the right question paper.
1) Answer five questions and at least one from each module.
2) Statistical tables are allowed
3) Make suitable assumptions wherever required.
4) Figures to the right indicate full marks.

MODULE - I

Q.1 a) Show that the diagonal element of a skew – symmetric matrix are all zero. **04**

b) Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$. **08**

c) Test for consistency and solve if possible **08**

$$2x - y + 3z - 2u = -5;$$
$$x + 5y - z + 3u = 1;$$
$$x - 2z + 2u = 9;$$
$$x + 2y - z - u = -5$$

Q.2 a) Prove that, if A and B are similar matrices then they have then same Eigen values. **03**

b) Find the Eigen values and Eigen vectors of the matrix **09**

$$A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}.$$

c) Verify the Cayley – Hamilton theorem for the matrix **08**

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}.$$

MODULE – II

Q.3 a) Show that $P(\bar{A}) = 1 - P(A)$, where A is an event in sample space S. **04**

b) The probability density function of a continuous random variable X is given by, **08**

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$$f(X) = \begin{cases} k(2x - x^2); for\ 0 < x < 2 \\ 0 \quad ; elesewhere. \end{cases}$$

Compute (i) Value of k (ii) $P[X < 1]$.

c) A company which manufactures syrup bottles, Finds that 0.1 percent of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug, manufacturer buys 100 boxes from the manufacturer of these bottles. Using poison distribution, find how many boxes will contain: (i) no defective bottles (ii) at least two defective bottles. **08**

Q.4 a) The lifetimes of certain kinds of electronic devices have a mean of 300 hours and standard deviation of 25 hours. Assuming that the distribution of these lifetimes, (measured to the nearest hour) can be approximated closely by a normal distribution, compute the following: **06**

i) $P(X \leq 300)$

ii) $P(220 < X < 260)$

b) Compute the coefficient of correlation for the following data: **08**

Marks in Physics	20	30	28	17	19	23	35	13	16	38
Marks in Mathematics	18	35	20	18	25	28	33	18	20	40

c) The percentage of A's given in a physics course at a certain university over a long period of time was 10%. During one particular term there were 40 A's in a group of 300 students. Test the significance of this result at the 0.05 level. **06**

MODULE – III

Q.5 a) If $L[f(t)] = F(s)$ then Prove the following **06**

i) $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(S)$

ii) $L[\int_0^t f(t)dt] = \frac{1}{s} F(s)$,

Where $L[f(t)]$ denotes the Laplace transform of f(t).

b) Find the Laplace transform of the following functions: **08**

(i) $f(t) = \sin h\ 4t - 3e^{-5t} + \cos 2t$

(ii) $g(t) = \sin 3t\ \cos 4t$

c) Find the Laplace transform of a square were given by **06**

$$f(t) = \begin{cases} K\ ; for\ 0 \leq t < a \\ -K\ ; for\ a \leq t < 2a \end{cases}$$

Given $f(t + 2a) = f(t)$.

Q.6 a) Find the inverse Laplace transform of the following functions; **08**

i) $F(s) = \frac{s^2 - 3s + 4}{s^3}$

ii) $G(s) = \frac{1}{s(s^2 + 4s + 5)}$

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b) Apply the convolution theorem to compute the inverse Laplace transform of the following function: **04**

$F(s) = \frac{1}{(s-1)(s-3)}$

c) Solve the equation $(D^2 + 4D + 13)y = e^{-t}\sin t$, given $y(0) = 0$ and $y'(0) = 0$ using Laplace transform. **08**

MODULE – IV

Q.7 a) If $F[f(x)] = F(s)$, then prove that **08**

i) $F[f(ax)] = \frac{1}{a}F\left(\frac{s}{a}\right)$

ii) $F[f(x-a)] = e^{-ias}F(s)$,

Where $F[f(x)]$ denotes the Fourier transform of $f(x)$.

b) Find the Fourier transform of the function $f(x) = \begin{cases} 1-|x|\,; if\ |x| < 1 \\ 0\,; \quad\quad if\ |x| > 1 \end{cases}$ **06**

c) Find Fourier sine transform of $f(x) = e^{-kx}, where\ x > 0\ and\ k > 0.$ **04**

Q.8 a) If $\mathbb{Z}[f(n)] = F(z)$, then prove that $\mathbb{Z}[f(n+1)] = z[F(z) - f(0)]$ where $Z[f(n)]$ denotes the Z – transform of $f(n)$. **04**

b) Find the inverse $\mathbb{Z}$ - transform of **08**

i) $F(z) = \frac{z^2}{(z-1)(z-2)}$

ii) $G(z) = \frac{2z+1}{(z-3)(z-4)}$

c) Solve the following difference equation, using $\mathbb{Z}$ - transform: **08**

$y_{n+2} - 5y_{n+1} + 6y_n = 4^n$ given $y_0 = 0$ and $y_1 = 1$

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